

I'm not robot!

As with my own course, the reader needs to have taken a first course in Probability; it also has rather more emphasis on deriving the mathematics than my course. Al-Khalil, a middle eastern mathematician, wrote the Book of Cryptographic Messages, which demonstrates the first use of permutation and combination to list all the Arabic words with or without vowels. This was the earliest form of probability and statistics. Probability of a branch of mathematics relating the numerical illustration of how likely an event can exist. The likelihood of any event to occur is a number between 0 and 1, where 0 indicates the impossibility of the event and 1 indicates certainty. Probability theory is widely used in the area of studies such as statistics, finance, gambling artificial intelligence, machine learning, computer science, game theory, and philosophy. A probability forecast is an assessment of how likely an event can occur in terms of percentage and record the risks associated with weather. Meteorologists around the world use different instruments and tools to predict weather changes. They collect the weather forecast database from around the world to estimate the temperature changes and probable weather conditions for a particular hour, day, week, and month. If there are 40 % chances of raining then the weather condition is such that 40 out of 100 days it has rained. Sports Strategies: In sports, analyses are conducted with the help of probability to understand the strengths and weaknesses of a particular team or player. Analysts use probability and odds to foretell outcomes regarding the team's performance and members in the sport. Coaches use probability as a tool to determine in what areas their team is strong enough and in which all areas they have to work to attain victory. Trainers even use probability to gauge the capacity of a particular player in his team and when to allow him to play and against whom. A cricket coach evaluates a player's batting and bowling capability by taking his average performances in previous matches before placing him in the lineup. Insurance: insurance companies use the theory of probability or theoretical probability for framing a policy or completing at a premium rate. The theory of probability is a statistical method used to predict the possibility of future outcomes. Issuing health insurance for an alcoholic person is likely more expensive compared to the one issued to a healthy person. Statistical analysis shows high health risks for a regular alcoholic person, ensuring them is a great financial risk given a higher probability of serious illness and hence filing a claim of premium money. In Games: Blackjack, poker, gambling, all sports, board games, video games use probability to know how likely a team or person has chances to win. When two dices are rolled simultaneously, the outcomes will be as given below Game Theory: It is the study of mathematical representation of strategic relations among analytical outcomes. It has applications in social science, logic, system science, and computer science. In 1944, John Von Neumann published a paper, "Theory of Games and Economic Behaviour". He proved Brouwer's fixed point theorem on continuous mapping into compact convex sets, the standard game theory method. Application of portability in Game theory : 1. Economics and business: Economists use game theory as a tool to analyze economic competition and phenomena such as bargaining, voting theory, auction, mechanism design. Executives, investors, and managers in the business world use the game theory strategy for investments, launching of new products, or entering a new business. Game theory models force each player to consider the action made by their competitor and plan the next strategy. 2. In politics: Diplomats and politicians use game theory to analyze any situation of conflict between individuals, companies, states, and political parties. It is also used in war strategies, political voting, and political affairs. 3. In philosophy: Philosophers use game theory in various aspects of philosophy. 4. In biology: It is applied to the analysis of the abnormal natural phenomenon in biology. © 1996-2014, Amazon.com, Inc. or its affiliates Is it a hat? Or an elephant eaten by a snake? Or a mix of normal distributions? Image by the author. Probability distributions are mathematical functions describing probabilities of things happening. Many processes taking place in the world around us can be described by a handful of distributions that have been well-researched and analyzed. Getting one's head around these few goes a long way towards being able to statistically model a range of phenomena. Let's take a look at six useful probability distributions! Binomial distribution Arguably the most intuitive yet powerful probability distribution is the binomial distribution. It can be used to model binary data, that is data that can only take two different values, think "yes" or "no". This makes the binomial distribution suitable for modeling decisions or other processes, such as Did the client buy the product, or not? Did the medicine help the patient to recover, or not? Was the online ad clicked on, or not? Two key elements of the binomial model are a set of trials (binary experiments) and the probability of success (success is an ad clicked on or a patient cured). For instance, the trials may consist of 10 coin flips — each of them is a binary experiment with two possible outcomes: heads or tails. The probability of success, defined as tossing heads, is 50%, assuming the coin is fair. The binomial distribution can answer the question: what is the probability of observing different numbers of heads in 10 tosses with a fair coin? Plotting the binomial PMF. Some plot-formatting code has been omitted. The probability of getting X heads in 10 fair coin tosses. Image by the author. The binomial probability mass function plotted above is the answer! You can see that it's most likely to observe 5 heads in 10 tosses, and the probability of such an outcome is roughly 25%. An example of practical usage of the binomial distribution is to model clicks vs non-clicks on an ad banner, where the probability of success is the click-through rate. The binomial distribution may serve as an anomaly detector. Say you have displayed your ad to 750 users and 34 clicked on it. This gives a click-through rate of 4.5%. If you know the average click-through rate for all your previous ads was 6%, you might want to ask: what's the probability of observing no more than 4.5% this time? To answer this, you can model your clicks with a binomial distribution with the success probability of 6%, which looks like this: The probability of getting X heads in 750 impressions assuming a click-through rate of 6%. Image by the author. So, what's the probability of observing no more than 34 clicks in 750 impressions? The answer is the sum of all bars from 0 through 34, or:  $\text{binom.cdf}(34, 750, 0.06)$  which is 4.88%, quite unlikely. Either the new ad is really bad, or something fishy is happening here... Are you sure your ad provider actually did display it to 750 users? The binomial distribution describes yes-no data. It may serve as an anomaly detector or a tool for Bayesian A/B testing. Another common application of the binomial distribution is for Bayesian A/B testing. Imagine you have run two marketing campaigns and want to compare them. A simple comparison of two click-through rates can be misleading, as there is a random component in both, and one can seem higher than the other as a result of chance alone. There are two ways to establish which campaign was better: you can either resort to classical hypothesis testing (more or this later), or explicitly estimate the probability that campaign A was better than campaign B with the Bayesian approach. The latter assumes that the clicks follow the binomial distribution and the calculation of the Bayesian posterior probabilities involves the binomial density formula. How to do this is a topic that deserves a separate treatment, but if you'd like to learn about Bayesian A/B testing or Bayesian statistics in general, I encourage you to take a look at my introductory article to the topic or at the Bayesian Data Analysis in Python course that I teach on DataCamp. Posterior click-through rates of two ad campaigns: A and B. In this case, B is better with a 93% probability. Picture from the Bayesian Data Analysis in Python course, taught by the author at DataCamp. Poisson distribution Another very useful distribution is the Poisson distribution. Just like the binomial, it is also a discrete distribution, meaning that it has a limited set of possible outcomes. For the binomial distribution, there were just two: yes or no. For the Poisson distribution, there might be more, but they can only be natural numbers: 0, 1, 2, and so on. The Poisson distribution can be used to describe events that occur at some rate over time or space. Such processes are everywhere: clients make a purchase in your online store every X minutes, and every Yth product coming off the assembly line is defective. The Poisson distribution has one parameter, typically denoted with a greek letter  $\lambda$  (lambda), which denotes the rate at which events are happening. The typical use case is to estimate  $\lambda$  from the data and then use the resulting distribution to perform queueing simulations that help allocate resources. Imagine you have a machine learning model deployed in the cloud and receiving requests from your customers in real-time. How many cloud resources do you need to pay for in order to be 99% sure you can serve all the traffic that arrives at the model in any one-minute period? To answer this question, you first need to know how many requests are coming, on average, in a one-minute period. Say that it's 3.3 requests on average based on your traffic data. That's your  $\lambda$ , and hence the Poisson distribution describing your data looks like this: Plotting the Poisson PMF. Some plot-formatting code has been omitted. The probability of X requests coming within a one-minute period, based on the average rate of 3.3. Image by the author. Based on your Poisson distribution, you can calculate the probability of observing 2 events within one minute:  $\text{poisson.pmf}(2, 3.3)$ , which yields 20%, or the probability of getting 5 requests or less:  $\text{poisson.cdf}(5, 3.3)$  which is 88%. Poisson distribution describes events that occur at some rate over time or space. It can be used to conduct queueing simulations that help allocate resources. Cool, but the question was: how many requests per minute do you need to be able to process so that you can be 99% sure to process all the traffic? One way to answer this is to simulate a lot (say, 1,000,000) one-minute events and calculate the 99th percentile of the number of requests. This yields 8, meaning that if you buy enough resources to process 8 requests per minute, you can be 99% sure to process all the traffic in any one-minute period. Exponential distribution Another distribution, closely related to Poisson, is the exponential distribution. If the number of events occurring in some time period follows the Poisson process, then the time between those events is described by the exponential distribution. Continuing the previous example, if we observe 3.3 requests per minute on average, we can model the time between the requests using the exponential distribution. Let's express it in seconds: 3.3 requests per minute is 0.055 requests per second. One caveat is that in Python's `scipy` package, the exponential distribution is parametrized with a scale, which is the inverse of a rate. Also, the exponential distribution is continuous rather than discrete, which means it can take infinitely many values. To emphasize this, we plot it as a shaded area rather than as vertical bars. The distribution in question looks as follows: Plotting the exponential PDF. Some plot-formatting code has been omitted. The probability of X minutes passing between two events, based on the average rate of 3.3 events per minute. Image by the author. With the average rate of more than three requests per minute, in most cases, there won't be more than a minute since one request before another one comes in. Sometimes, however, as many as five minutes may pass requestlessly! Just like the binomial distribution, the exponential distribution can also serve as an anomaly detector. After you haven't seen a request for 5 minutes, you might start to wonder whether your systems are still alive. What's the probability of five or more minutes passing without a request? That's the sum of all bars starting from five and to the right up to infinity, which is equivalent to one minus the bars to the left from five since the sum of all bars must be one. So, it's  $1 - \text{expn.cdf}(4, 1/3.3)$ , which is 2.5%. Not very likely, but can happen once in a while. Exponential distribution can be used as an anomaly detector or as a simple benchmark for predictive models. Another common use case for the exponential distribution is as a simple benchmark for predictive models. For example, in order to predict when each product will be bought again, you might use gradient boosting or build a fancy neural network. To evaluate their performance, it's a good practice to compare them against a simple benchmark. If you state your problem as predicting the time until the next purchase, then the exponential distribution is a great benchmark. Once you have the training data (days till next purchase = [1, 4, 2, 2, 3, 6, ..., ]), all you have to do is to fit the exponential distribution to these data and take its mean as the prediction:  $\text{avg.} = \text{expn.fit}(\text{days till next purchase})$ . Normal distribution The normal distribution, also known as the bell-curve, is perhaps the most famous one, and also the most widely used — although often implicitly. First and foremost, the Central Limit Theorem, which is the cornerstone of statistical inference, is all about the normal distribution. I encourage you to read more about it here: In a nutshell, the CLT establishes that some statistics, such as a sum or a mean, calculated from random samples of data follow the normal distribution. The two most important practical use cases for the CLT are regression analysis and hypothesis testing. First, let's talk regression. In a regression model, the dependent variable is being explained by some predictor variables plus an error term, which we assume to be normally distributed. This assumed normality of error stems from the CLT: we can view the error as the sum of many independent errors caused by omitting important predictors or by random chance. Each of these many errors can have any distribution, but their sum will be approximately normal by the CLT. Assumption of errors normality in regression models can be justified by the Central Limit Theorem. It is also useful for some hypothesis tests. Second, hypothesis testing. More elaborate examples will follow shortly when we talk about the chi-square and F distributions. For now, consider this short example: a company specializing in ads retargeting boasts that their precisely targeted ads receive a click-through rate of 20% on average. You run a pilot of 1000 impressions with them and observe 160 clicks, which means the click-through rate of only 16%. Is the targeting not as good, or was it just bad luck? If you consider the 1000 users to whom to ads were displayed a random subset from the larger population of users, then the CLT kicks in and the click-through rate should follow a normal distribution. Assuming the targeting company tells the truth, it should be a normal with a mean of 0.2 and a standard deviation of 0.013. Check out my post on the CLT for the calculations, or just trust me this. The distribution in question looks as follows: Reference distribution of the CTR versus the observed CTR. Generated by the author. Under this distribution, the 16% click-through rate you have observed is rather improbable, indicating that the alleged 20% average is most likely not true. Chi-square distribution The  $\chi^2$  or chi-square distribution comes in handy in A/B/C testing. Imagine you have conducted an experiment in which you have randomly displayed your website in different color versions to the users. You're curious which color leads to the most purchases made through the website. Each color has been displayed to 1000 users, and here are the results: Website color experiment results. Image by the author. The yellow version seems to be almost twice as good as the blue one, but the numbers are low, so how can you be sure these results are not due to chance? Hypothesis testing to the rescue! To learn more about the topic, don't hesitate to take a detour to my Hypothesis Tester's Guide: Here, let me just highlight the role of the chi-square distribution in the process. The first step is to assume that the three website versions generate the same number of purchases, on average. If that were to be true, we would expect the same number of purchases for each version, and it would be  $(17+9+14)/3$ , or 13.33 purchases. The chi-square distribution allows us to measure how much the observed purchases depart from what's expected if the color made no difference. Chi-square distribution is useful for A/B/C testing. It measures how likely it is that the experimental results we got are a result of chance alone. It turns out that if we compute the (appropriately scaled and then squared) difference between the expected and observed values for each color, we arrive at a test statistic that follows the chi-square distribution! We can then infer, based on the p-value, whether the variation in the number of purchases we've observed was due to chance, or rather indeed due to the color difference. This so-called chi-square test is a one-liner in Python: Chi-square test for the website color experiment. We get a chi-square (chisq) value of 2.48 and the distribution is parametrized with 2 degrees of freedom (df). Hence, the distribution in question looks like this: Plotting the chi-square PDF with 2 degrees of freedom. Some plot-formatting code has been omitted. The chi-square distribution with 2 degrees of freedom. Image by the author. The chi-square value of 2.48 seems pretty likely under this distribution, which leads us to conclude that the differences in the number of purchases for differently-colored websites can be caused by random chance alone. This is further proved by the high p-value of almost 0.3. Please check out the HTG for more on hypothesis testing. F-distribution Let's stick to the website color experiment, an example of A/B/C testing. We have assessed the impact of color on a discrete variable: the number of purchases. Let's consider a continuous variable now, such as the number of seconds spent, on average, on each website variant. As part of the experiment, we have randomly shown the users different website variants and we calculated the time they spent on the website. Below are the data for 15 users (five were shown the yellow version, another five the blue one, and the last five the green one). For such an experiment to be valid, we would need many more than 15 users, this is just a demonstration. Seconds spent on different color variants of the website. Image by the author. We are interested in the extent to which the differences between the means are larger than what might have been produced by random chance. If they are significantly larger, then we might conclude that the color indeed impacts the time spent on the website. Such a comparison between means is referred to as ANOVA, or analysis of variance. The inference is quite similar to the one we've already discussed when talking about the chi-square distribution. We calculate the test statistic as the ratio between the variability among the group means and the variation within the groups. Such a statistic follows a distribution called the F-distribution. Knowing this, we can use the p-value to reject (or not) the assumption that the means are different due to chance alone. F-distribution is used for A/B/C testing when the outcome we measure is continuous. In Python, we can run ANOVA using the `statsmodels` package. The internal computations are based on a linear regression model, hence the call to `ols()` and gathering the data into a two-column data frame. Analysis of variance (ANOVA) for the impact of website color on the time spent on it. And here is the ANOVA table that we got: ANOVA output. Our F-distribution is parametrized with two degrees-of-freedom parameters: one for differences between the color-means and the grand average (2) and another for the differences within each color (12). Hence, our distribution looks as follows: Plotting the F PDF with 2 and 12 degrees of freedom. Some plot-formatting code has been omitted. The F-distribution with 2 and 12 degrees of freedom. Image by the author. Since we got the F-statistic equal to 7.6 (and, consequently, because the p-value is small), we conclude that the difference between the mean time spent on different website variants could not have been caused by random chance. The color does play a role here! Recap Binomial distribution is useful for modeling yes-no data. It can serve as an anomaly detector or a tool for Bayesian A/B testing. Poisson distribution can be used to describe events that occur at some rate over time or space. It is often used to conduct queueing simulations that help allocate resources. Exponential distribution describes that the time between two events following the Poisson distribution. It can be used as an anomaly detector or as a simple benchmark for predictive models. Normal distribution describes some statistics computed from random data samples, as established by the Central Limit Theorem. Thanks to this, we can assume the normality of errors in regression models, or conduct some hypothesis tests easily. Chi-square distribution is typically used for A/B/C testing. It measures how likely it is that the experimental results we got are a result of chance alone. F-distribution is used for A/B/C testing when the outcome we measure is continuous, e.g. in the ANOVA analysis. The inference is similar to the one using chi-square for discrete outcomes. Thanks for reading! If you liked this post, why don't you subscribe for email updates on my new articles? And by becoming a Medium member, you can support my writing and get unlimited access to all stories by other authors and myself. Need consulting? You can ask me anything or book me for a 1:1 here. You can also try one of my other articles. Can't choose? Pick one of these:





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